

# CP violation in B decays within QCD factorization<sup>\*</sup>

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**Abstract.** We analyze the extraction of weak phases from CP violation in  $B \rightarrow \pi^+\pi^-$  decays. By combining the information on mixing induced CP violation in  $B \rightarrow \pi^+\pi^-$ , namely  $S_{\pi\pi}$ , with the precision observable  $\sin 2\beta$  obtained from the “gold-plated” mode  $B \rightarrow \psi K_S$ , we propose the determination of the unitarity triangle. We also discuss alternative ways to analyze  $S_{\pi\pi}$  which can be useful if new physics affects  $B_d-\bar{B}_d$  mixing. Predictions and uncertainties for  $r$  and  $\phi$  in QCD factorization are examined in detail. It is pointed out that a simultaneous expansion in  $1/m_b$  and  $1/N_C$  leads to interesting simplifications. At first order infrared divergences are absent, while the most important effects are retained. Independent experimental tests of the factorization framework are briefly discussed.

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## 1 Introduction

The main goal of the current experimental program at the SLAC and KEK  $B$ -meson factories is a stringent test of the standard model description of CP violation. In the future this aim will be pursued with higher precision measurements from hadron machines at Fermilab and CERN.

A crucial benchmark is the time-dependent CP violation in  $B \rightarrow \psi K_S$  decays, which allows us to infer the CKM phase  $\beta$  with negligible hadronic uncertainties. Likewise of central importance for obtaining additional information on CKM parameters is the time-dependent CP violation, both mixing-induced ( $S_{\pi\pi}$ ) and direct ( $C_{\pi\pi}$ ), in  $B \rightarrow \pi^+\pi^-$ . However, in this case the extraction of weak phases is complicated by the so-called penguin pollution, leading to a hadronic model dependent estimate of the CP asymmetries in  $B \rightarrow \pi^+\pi^-$ . A possible strategy to circumvent this problem is the use of symmetry argument [1–6], such as the isospin or the SU(3) symmetry. However these methods are actually very limited which is likely to prevent a successful realization.

In this talk, we present the result of [7], where a new way of exploring informations from the mixing induced CP violation parameter  $S_{\pi\pi}$  in the  $B \rightarrow \pi^+\pi^-$  mode, combined with the precision observable  $\sin 2\beta$  was suggested in order to explore the CKM unitarity triangle. Our estimate of the penguin parameters are carried out in QCD factorization and confronted to other independent estimates.

## 2 Basic formulas

The time-dependent CP asymmetry in  $B \rightarrow \pi^+\pi^-$  decays is defined by

$$A_{CP}^{\pi\pi}(t) = \frac{B(B(t) \rightarrow \pi^+\pi^-) - B(\bar{B}(t) \rightarrow \pi^+\pi^-)}{B(B(t) \rightarrow \pi^+\pi^-) + B(\bar{B}(t) \rightarrow \pi^+\pi^-)},$$

$$= -S_{\pi\pi} \sin(\Delta m_B t) + C_{\pi\pi} \cos(\Delta m_B t), \quad (1)$$

where

$$S_{\pi\pi} = \frac{2 \operatorname{Im} \xi}{1 + |\xi|^2}, \quad C_{\pi\pi} = \frac{1 - |\xi|^2}{1 + |\xi|^2}, \quad \xi = e^{-2i\beta} \frac{e^{-i\gamma} + P/T}{e^{+i\gamma} + P/T}. \quad (2)$$

In terms of the Wolfenstein parameters  $\bar{\rho}$  and  $\bar{\eta}$  the CKM phase factors read

$$e^{\pm i\gamma} = \frac{\bar{\rho} \pm i\bar{\eta}}{\sqrt{\bar{\rho}^2 + \bar{\eta}^2}}, \quad e^{-2i\beta} = \frac{(1 - \bar{\rho})^2 - \bar{\eta}^2 - 2i\bar{\eta}(1 - \bar{\rho})}{(1 - \bar{\rho})^2 + \bar{\eta}^2}. \quad (3)$$

The penguin-to-tree ratio  $P/T$  can be written as  $P/T = r e^{i\phi} / \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ . The real parameters  $r$  and  $\phi$  defined in this way are pure strong interaction quantities without further dependence on CKM variables.

For any given values of  $r$  and  $\phi$  a measurement of  $S_{\pi\pi}$  defines a curve in the  $(\bar{\rho}, \bar{\eta})$ -plane. Using the relations above this constraint is given by the equation

$$S_{\pi\pi} = \frac{2\bar{\eta}[\bar{\rho}^2 + \bar{\eta}^2 - r^2 - \bar{\rho}(1 - r^2) + (\bar{\rho}^2 + \bar{\eta}^2 - 1)r \cos \phi]}{((1 - \bar{\rho})^2 + \bar{\eta}^2)(\bar{\rho}^2 + \bar{\eta}^2 + r^2 + 2r\bar{\rho} \cos \phi)} \quad (4)$$

Similarly the relation between  $C_{\pi\pi}$  and  $\bar{\rho}, \bar{\eta}$  is straightforward. The current experimental results for  $S_{\pi\pi}$  and  $C_{\pi\pi}$

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are

$$\begin{array}{cc} \text{(BaBar[8])} & \text{(Belle[9])} \\ S_{\pi\pi} = +0.02 \pm 0.34 \pm 0.05 & -1.23 \pm 0.41_{-0.07}^{+0.08} \\ C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04 & -0.77 \pm 0.27 \pm 0.08 \end{array}$$

A recent preliminary update from BaBar [10] gives  $S_{\pi\pi} = -0.40 \pm 0.22 \pm 0.03$ , and  $C_{\pi\pi} = -0.19 \pm 0.19 \pm 0.05$ .

The penguin parameter  $r e^{i\phi}$  has been computed in [11] in the framework of QCD factorization. The result can be expressed in the form

$$r e^{i\phi} = -\frac{a_4^c + r_\chi^\pi a_6^c + r_A[b_3 + 2b_4]}{a_1 + a_4^u + r_\chi^\pi a_6^u + r_A[b_1 + b_3 + 2b_4]}, \quad (5)$$

where we neglected the very small effects from electroweak penguin operators. A recent analysis gives [7,12]

$$r = 0.107 \pm 0.031, \quad \phi = 0.15 \pm 0.25, \quad (6)$$

where the error includes an estimate of potentially important power corrections. In order to obtain additional insight into the structure of hadronic  $B$ -decay amplitudes, it will be also interesting to extract these quantities from other  $B$ -channels, or using other methods. In this perspective, we have considered them in a simultaneous expansion in  $1/m_b$  and  $1/N_C$  ( $N_C$  is the number of colours).

As stated above, the most important contributions in  $(r, \phi)$  are the factorization coefficients  $a_{1,4,6}$  and the weak annihilations ones  $b_{1,3,4}$  as shown<sup>1</sup> in (5). Expanding these coefficients to first order in  $1/m_b$  and  $1/N_C$  we find

$$a_1 \doteq C_1 + \frac{C_2}{N_C} \left[ 1 + \frac{C_F \alpha_s}{4\pi} V_\pi \right] + \frac{C_2}{N_C} \frac{C_F \pi \alpha_s}{N_C} H_{\pi\pi,2}, \quad (7)$$

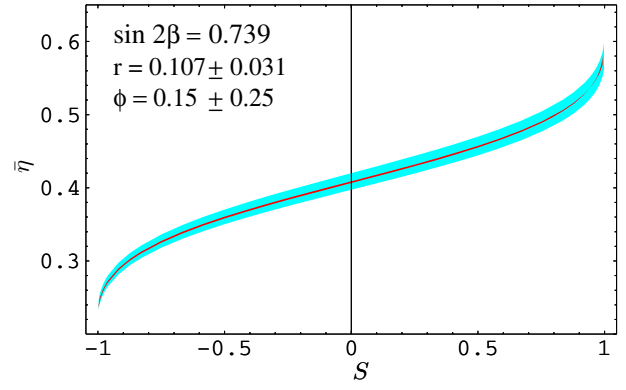
$$a_4^p \doteq C_4 + \frac{C_F \alpha_s}{4\pi} \frac{P_{\pi,2}^p}{N_C}, \quad r_\chi^\pi a_6^p \doteq r_\chi^\pi C_6, \quad b_{1,3,4} \doteq 0,$$

where  $H_{\pi\pi,2}$  is the leading-twist effect in the hard spectator scattering. We observe that to this order in the double expansion, the uncalculable power  $H_{\pi\pi,3}(\sim X_H)$  does not appear in  $a_1$ , to which it only contributes at order  $1/m_b N_C$ . Using our default input parameters, one obtains the central value [7]:  $(r_{N_C}, \phi_{N_C}) = (0.084, 0.065)$ , which seems to be in a good agreement with the standard QCD factorization framework at the next-to-leading order.

As a second cross-check, one can extract  $r$  and  $\phi$  from  $B^+ \rightarrow \pi^+ \pi^0$  and  $B^+ \rightarrow \pi^+ K^0$ , leading to the central value [7]  $(r_{SU3}, \phi_{SU3}) = (0.081, 0.17)$ , in agreement with the above results<sup>2</sup>, although their definitions differ slightly from  $(r, \phi)$  (see [7] for further discussions).

<sup>1</sup> both corrections depend on the unknown power corrections effects, described by phenomenological quantities,  $X_{H,A} = (1 + \rho_{H,A} e^{i\phi_{H,A}}) \ln \frac{m_B}{\Lambda_h}$ .

<sup>2</sup> one can compare also the  $r_{SU3}$  to its experimental value  $r_{SU3}^{exp} = 0.099 \pm 0.014$ .



**Fig. 1.** CKM phase  $\bar{\eta}$  as a function of  $S_{\pi\pi}$ . The *dark (light)* band reflects the theoretical uncertainty in the parameter  $\phi$  ( $r$ )

### 3 Exploring the unitarity triangle in the SM and beyond

In this section we discuss the determination of the unitarity triangle by combining the information from  $S_{\pi\pi}$  with the value of  $\sin 2\beta$ , well known from the “gold-plated” mode  $B \rightarrow J/\Psi K_S$ . The angle  $\beta$  of the unitarity triangle is given by

$$\tau \equiv \cot \beta = \sin 2\beta \left( 1 - \sqrt{1 - \sin^2 2\beta} \right)^{-1}. \quad (8)$$

The current world average [10]

$$\sin 2\beta = 0.739 \pm 0.048, \quad (9)$$

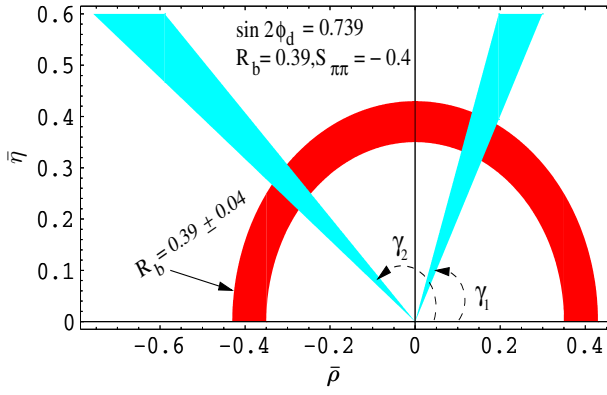
implies  $\tau = 2.26 \pm 0.22$ . Given a value of  $\tau$ ,  $\bar{\rho}$  is related to  $\bar{\eta}$  by  $\bar{\rho} = 1 - \tau \bar{\eta}$ . The parameter  $\bar{\rho}$  may thus be eliminated from  $S_{\pi\pi}$  in (4), which can be solved for  $\bar{\eta}$  to yield

$$\bar{\eta} = \frac{1}{(1 + \tau^2) S_{\pi\pi}} \left[ (1 + \tau S_{\pi\pi})(1 + r \cos \phi) - \sqrt{(1 - S_{\pi\pi}^2)(1 + r^2 + 2r \cos \phi) - (1 + \tau S_{\pi\pi})^2 r^2 \sin^2 \phi} \right]. \quad (10)$$

The two observables  $\tau$  (or  $\sin 2\beta$ ) and  $S_{\pi\pi}$  determine  $\bar{\eta}$  and  $\bar{\rho}$  once the theoretical penguin parameters  $r$  and  $\phi$  are provided. It is at this point that some theoretical input is necessary. We will now consider the impact of the parameters  $r$  and  $\phi$ , and of their uncertainties, on the analysis.

We first would like to point out that the sensitivity of  $\bar{\eta}$  in (10) on the strong phase  $\phi$  is rather mild. In fact, the dependence on  $\phi$  enters in (10) only at second order, and hence suppressed for small  $\phi$ . This nice feature is *la bienvvenue* because the estimate of the strong phase is difficult.

The determination of  $\bar{\eta}$  as a function of  $S_{\pi\pi}$  is shown in Fig. 1, which displays the theoretical uncertainty from the penguin parameters  $r$  and  $\phi$  in QCD factorization.



**Fig. 2.** Determination of the unitarity triangle from  $S_{\pi\pi}$ ,  $R_b$ ,  $\sin 2\phi_d$ , and the penguin parameters as in Fig. 1. The resulting ranges of the CKM angle  $\gamma$  are  $\gamma_1 = 67^\circ \pm 4^\circ$  and  $\gamma_2 = 138^\circ \pm 4^\circ$ . The *light bands* reflect the uncertainty in  $r$  and  $\phi$

In the determination of  $\bar{\eta}$  and  $\bar{\rho}$  described here discrete ambiguities do in principle arise, however they are ruled out using the standard fit of the unitarity triangle (see [7] for further discussions).

Up to now, our analysis was carried out within the standard model. However, in the presence of new physics this may no longer be valid. To be specific we shall assume the plausible scenario where the new physics contributions modify the phase of  $B_d$ - $\bar{B}_d$  mixing  $\phi_d$ , whereas the  $B$  decay amplitudes remain unchanged. The CP asymmetry in  $B \rightarrow J/\Psi K_S$  (9) must then be interpreted as the quantity  $\sin 2\phi_d$ . Since we can no longer relate  $\sin 2\phi_d$  to  $\bar{\rho}$  and  $\bar{\eta}$ , we should fix it to the experimental value in (9) when using (2), where  $\beta$  is to be replaced by  $\phi_d$ . A similar analysis has already been carried out in [14].

Writing  $(\bar{\rho}, \bar{\eta}) = R_b(\cos \gamma, \sin \gamma)$  and  $R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ , we can express  $S_{\pi\pi}$  as

$$S_{\pi\pi} = \text{Im} \left[ \frac{e^{-2i\phi_d} (R_b \cos \gamma + r \cos \phi - iR_b \sin \gamma)^2 + \kappa^2}{(R_b \cos \gamma + r \cos \phi)^2 + R_b^2 \sin^2 \gamma + \kappa^2} \right], \quad (11)$$

where  $\kappa = r \sin \phi$ . From this relation, for given values of  $r$  and  $\phi$ , and using the experimental results for  $S_{\pi\pi}$ ,  $\sin 2\phi_d$  and  $R_b$ ,  $\gamma$  can be determined and hence  $\bar{\rho}$  and  $\bar{\eta}$ .

Experimentally one has [10,13]  $\sin 2\phi_d = 0.739 \pm 0.048$  and  $R_b = 0.39 \pm 0.04$ . As emphasized in [14], there is a discrete ambiguity in the sign of  $\cos 2\phi_d$ , which yields two different solutions for  $\gamma$ . The larger value of  $\gamma$  will be obtained for negative  $\cos 2\phi_d$ . The analysis is represented in Fig. 2, assuming a particular scenario for illustration and displaying the impact of the theoretical uncertainty in  $r$  and  $\phi$ .

## 4 Summary

In this talk, we have proposed strategies to extract information on weak phases from CP violation observables in

$B \rightarrow \pi^+\pi^-$  decays even in the presence of hadronic contributions related to penguin amplitudes. Assuming knowledge of the penguin pollution, an efficient use of mixing-induced CP violation in  $B \rightarrow \pi^+\pi^-$  decays, measured by  $S_{\pi\pi}$ , can be made by combining it with the corresponding observable from  $B \rightarrow \psi K_S$ ,  $\sin 2\beta$ , to obtain the unitarity triangle parameters  $\bar{\rho}$  and  $\bar{\eta}$ . The sensitivity on the hadronic quantities, which have typical values  $r \approx 0.1$ ,  $\phi \approx 0.2$ , is very weak. In particular, there are no first-order corrections in  $\phi$ . For moderate values of  $\phi$  its effect is negligible. An alternative analysis of  $S_{\pi\pi}$  especially suitable for the presence of a new-physics phase in  $B_d$ - $\bar{B}_d$  mixing was discussed.

Concerning our penguin parameters, namely  $r$  and  $\phi$ , they were investigated systematically within the QCD factorization framework. To validate our theoretical predictions, we have calculate these parameters in the  $1/m_b$  and  $1/N_C$  expansion, which exhibits a good framework to control the uncalculable power corrections, namely  $X_{H,A}$ , in the factorization formalism. As an alternative proposition, we have also considered to extract  $r$  and  $\phi$  from other  $B$  decay channels, such as  $B^+ \rightarrow \pi^+\pi^0$  and  $B^+ \rightarrow \pi^+K^0$ , relying on the SU(3) argument. Using these three different approaches, we found a compatible picture in estimating these hadronic parameters.

Finally, we hope that in the forthcoming experimental measurements of CP violation in  $B \rightarrow \pi^+\pi^-$  decays, our analysis will be useful in achieving a reliable control over these unknown penguin parameters.

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